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Financial Distress and the Cross-section of Equity Returns

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ABSTRACT

We explicitly consider financial leverage in a simple equity valuation model and study the cross-sectional implications of potential shareholder recovery upon resolution of financial distress. Our model is capable of simultaneously explaining lower returns for financially distressed stocks, stronger book-to-market effects for firms with high default likelihood, and the concentration of momentum profits among low credit quality firms. The model further predicts (i) a hump-shaped relationship between value premium and default probability, and (ii) stronger momentum profits for nearly distressed firms with significant prospects for shareholder recovery. Our empirical analysis strongly confirms these novel predictions.

Financial distress is frequently invoked to justify the existence of "anomalous" cross-sectional properties of equity returns such as the size effect and the value premium (e.g., Fama and French (1996)). The existing empirical evidence, however, presents a complex picture that eludes a coherent and unifying explanation. Griffin and Lemmon (2002) and Vassalou and Xing (2004) show that the book-to-market and size effects are concentrated in high default risk firms, thus lending credence to the conjecture that the value and

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size effects are closely related to distress risk. However, Dichev (1998) and Campbell, Hilscher, and Szilagyi (2008) document that high default probability firms tend to have *lower* future stock returns, hence casting doubt on the notion of a market premium for distress risk. Furthermore, recent work by Avramov et al. (2007) indicates that profits of momentum strategies that buy "winners" and sell "losers" are remarkably concentrated among a small subset of firms with low credit ratings, which adds a new dimension to the complex relationship between financial distress and cross-sectional properties of equity returns.

In this paper, we show that these seemingly incongruent empirical patterns can be understood within an equity valuation model that explicitly accounts for financial leverage and recognizes that shareholders, by strategically defaulting on their debt, may recover part of the residual firm value upon the resolution of financial distress.[1] The resolution of financial distress includes debt restructuring and debt–equity exchange that do not necessarily lead to formal bankruptcy filings. It is therefore important to note that *shareholder recovery* is a broader concept than that of "violation of absolute priority" in bankruptcy proceedings. In a recent study, Morellec, Nikolov, and Schürhoff (2008) estimate that, among U.S. firms over the period from 1992 to 2004, the average shareholder recovery is about 20% of the asset value at the time of financial distress. Our work demonstrates the pervasive effects of shareholder recovery on the cross-sectional properties of equity returns.

We first develop our main intuition in a simple model in which we take capital structure and investment decisions as given. We then verify the robustness of our intuition in a more general model in which firms endogenously make investment and financing decisions based on their existing capital and debt levels. As in Berk, Green, and Naik (1999) and several more recent papers,[2] in our model equity beta is linked to firm characteristics such as the book-to-market ratio. The explicit inclusion of financial leverage allows us to show how leverage amplifies the book-to-market effect, thus providing a rationale for the findings of Griffin and Lemmon (2002) and Vassalou and Xing (2004), who document a stronger book-to-market effect in highly levered stocks.

More important, we show that the likelihood of shareholder recovery from firms in financial distress, a feature largely ignored in the previous asset pricing literature,[3] can fundamentally alter the riskiness of equity as default probability rises. All else being equal, at low levels of default probability, higher leverage increases equity beta. At high levels of default probability, however,

[1] Financial distress may result in missed payments, modified terms and structure of debt in private workouts, and ultimately, bankruptcy filings. In this paper, we use the terms "default" and "financial distress" interchangeably.

[2] The related literature includes Gomes, Kogan, and Zhang (2003), Carlson, Fisher, and Giammarino (2004), Zhang (2005), Cooper (2006), Gala (2006), Sagi and Seasholes (2007), and Novy-Marx (2008).

[3] Exceptions are Fan and Sundaresan (2000), who use this mechanism to study corporate bond spreads, and Garlappi, Shu, and Yan (2008), who adopt this feature to explain the negative relationship between default probability and stock returns.

the possibility of debt renegotiation and subsequent asset redistribution upon financial distress actually de-levers the equity beta and thus reduces the risk of equity. As a consequence, in the presence of shareholder recovery, our model predicts that equity beta and expected returns are hump shaped in default probability.

The hump-shaped relationship between expected returns and default probability is capable of simultaneously explaining two known empirical regularities: the inverse relationship between expected returns and default probability (Dichev (1998), Campbell et al. (2008), Garlappi et al. (2008), and George and Hwang (2010)), and the concentration of momentum profits in low credit quality stocks (Avramov et al. (2007)). We further show that this hump-shaped relationship has novel predictions for the cross-sectional properties of both value premium and momentum profits. Specifically, (i) in the presence of shareholder recovery the value premium is hump shaped in default probability; and (ii) among high default probability firms, momentum profits are larger for stocks with higher expected shareholder recovery.

The intuition for the pattern of value premium with respect to default probability is as follows. Consider two identical firms, A and B. Suppose firm A experiences a positive shock to its stock price and firm B a negative shock. As a consequence, firm A will have a smaller book-to-market ratio and a smaller default probability than B. If we construct a portfolio that is long B and short A, then the expected return on this portfolio, $ER_B - ER_A$, will depend crucially on the relationship between expected returns and default probability. If this relationship is monotonically increasing, as is the case when shareholders of both firms do not expect any recovery upon financial distress, then the spread $ER_B - ER_A$ is always positive. In contrast, if the relationship between expected returns and default probability is hump shaped, the sign of the spread will depend on the location of these two firms on the default probability spectrum. For low levels of default probability, expected returns are increasing in default probability and thus the spread $ER_B - ER_A$ is positive. However, for high levels of default probability, expected returns are decreasing in default probability and hence the spread is negative. This simple argument suggests a value spread that is hump shaped in default probability in the presence of possible shareholder recovery upon financial distress.

A similar argument can be made to justify our prediction regarding the relative strength of momentum profits. When the relationship between expected returns and default probability is hump shaped, a shock to prices—as reflected in the realized return—can have different implications for expected future returns. When the probability of default is low, a negative shock to the stock price (low realized return) increases the default probability and hence leads to a higher expected return. This results in a negative autocorrelation in returns. In contrast, when the probability of default is high, a negative shock to the stock price leads to a lower expected return and hence a positive autocorrelation in returns. Our model thus implies that return continuation should be more pronounced among firms with high default probability, a prediction that finds empirical support in Avramov et al. (2007). Moreover, our theory suggests that it is not financial distress per se that causes momentum strategies to be more profitable. Rather, momentum profits increase with the prospect of shareholder recovery among nearly distressed firms.

To properly interpret our analysis of momentum strategies, it is important to keep in mind that momentum is not equivalent to positive autocorrelation in returns, as pointed out by Lewellen (2002). The profitability of momentum strategies is mostly a cross-sectional phenomenon, which may have a number of plausible explanations, including behavioral and liquidity-based ones.[4] Our emphasis is on "enhanced momentum strategies," that is, momentum strategies that focus on a set of stocks whose returns exhibit positive autocorrelations, similar to those studied by Sagi and Seasholes (2007). However, unlike theirs, our model generates economically significant enhanced momentum profits through financial leverage and shareholder recovery upon financial distress instead of relying on growth options.

Several recent empirical studies provide evidence consistent with the implications of our theory. Favara, Schroth, and Valta (2010) exploit the exogenous variation in creditor protection provisions across countries and find that equity beta increases with the degree of credit protection and decreases with shareholder recovery. O'Doherty (2009) argues that the effect of financial distress on stock returns is consistent with the conditional CAPM. In addition, Zhang (2010) shows that the effect of shareholder recovery is particularly strong among firms with private debt, which is more conducive to private workouts upon financial distress.

In our own empirical analysis, we use as a measure of default probability the market-based *expected default frequency (EDF)*, acquired directly from Moody's KMV (Kealhofer, McQuown and Vasicek, MKMV hereafter). Our data set, available at the monthly frequency, spans the 1969 to 2007 period. As a preliminary step in our investigation, we construct conditional beta at the monthly frequency from short-window regressions on daily returns and confirm the existence of a pervasive hump shape in the relationship between conditional beta and default probability. The novel contribution of this paper is to recognize that this pattern of betas has implications for the variation of value premium and momentum profits across the spectrum of default probabilities. The bulk of our empirical analysis is thus focused on documenting this link and verifying the predictions of our theory on the cross-sectional properties of these two anomalies.

To verify our prediction of a hump-shaped relationship between value premium and default probability, we form portfolios of stocks sorted on book-to-market ratios and default probability. Contrary to Griffin and Lemmon (2002) and Vassalou and Xing (2004), our results indicate that the value premium is hump shaped instead of monotonically increasing in default probability: it increases when levels of EDF are low and declines sharply at very high levels of EDF. This hump-shaped pattern is robust to traditional risk adjustment procedures that account for market, size, book-to-market, momentum, and liquidity

[4] See, for example, Daniel, Hirshleifer, and Subrahmanyam (1998) and Asness, Moskowitz, and Pedersen (2009).

factors. We demonstrate that the discrepancy between our results and those previously documented in the literature stems from the sample selection and portfolio formation procedures.

To examine our conjecture on the relative strength of momentum profits across different levels of shareholder recovery for firms with high levels of default probability, we refine Jegadeesh and Titman's (1993) momentum profits by forming portfolios according to default probability and proxies for shareholder recovery, such as asset value, R&D expenditure, and degree of industry concentration. The results are strongly supportive of our theory. At high levels of default probability, momentum profits are considerably stronger when shareholder recovery is high. In contrast, shareholder recovery does not play a significant role at low levels of default probability. We further revisit the analysis of Avramov et al. (2007) using our broader data set and confirm that momentum profits are stronger in stocks with higher default probability. In particular, after adjusting for traditional risk factors, the enhanced momentum profits are significantly positive only among firms that rank in top EDF quintiles. Finally, we find that momentum profits load positively on the size factor at low levels of default probability but negatively at high levels of default probability. This indicates that at low leverage levels, the enhanced momentum profits likely originate from small firms with growth opportunities, as suggested by Sagi and Seasholes (2007), whereas at high leverage levels, the enhanced momentum profits most likely come from potential shareholder recovery, a feature often associated with large firms.

The rest of the paper proceeds as follows. In Section I we present a simple valuation model for levered equity and develop the main intuition for the effect of shareholder recovery on equity beta, the value premium, and momentum profits. In Section II we use data on default probabilities from MKMV to empirically confirm our predictions on the links between the value premium, momentum profits, and default probability. Section III concludes. The Appendix contains some key proofs related to equity valuation and default probability. The Internet Appendix contains all remaining proofs, the general version of the model in Section I that accounts for optimal capital structure decisions and growth options, and details about the general model's numerical analysis.[5]

I. A Simple Model of Levered Equity Returns

In this section we construct a stylized valuation model of levered equity in order to develop the main economic intuition underlying the connection between financial distress and cross-sectional properties of equity returns. To keep the analysis as simple as possible, we take a firm's capital structure as given and ignore growth options and financing frictions. In the Internet Appendix we generalize this setup by allowing for endogenous investment and financing decisions at the firm level over the business cycle.

[5] An Internet Appendix for this article is available online in the "Supplements and Datasets" section at http://www.afajof.org/supplements.asp.

A. The Firm

We consider a representative firm producing one unit of output per period of time. The real price of output at time t is e^{p_t}, where the log price p_t is assumed to follow a mean-reverting (Ornstein–Uhlenbeck (OU)) process

$$dp_t = (\mu^{\boldsymbol{P}} - \zeta p_t)\,dt + \sigma dW_t^{\boldsymbol{P}}, \tag{1}$$

with $\mu^{\boldsymbol{P}}$, ζ, and σ being firm-specific constants, and $dW_t^{\boldsymbol{P}}$ the increment of a standard Brownian motion under the physical measure $\boldsymbol{P}$. When the degree of mean reversion ζ goes to zero, process (1) collapses to a Brownian motion. The mean reversion case is more realistic when dealing with competition in the product market and relates more closely to the general model that we discuss in the Internet Appendix. The Brownian motion case, however, has the advantage of analytical tractability. As we will show below, the main intuition of the model is nevertheless unaffected by the nature of the process describing the output price.

The production of output requires an operating cost of c per unit of time. The firm finances its operations through a perpetual bond that pays a constant coupon of l per unit of time. The profit after interest service is thus $e^{p_t} - c - l$, which accrues to equity holders as long as the firm is operating.

When the firm is in financial distress, that is, when shareholders either enter into strategic renegotiation with debt holders or file for bankruptcy, we assume that equity holders can recover a fraction $\eta \in [0, 1]$ of the firm's residual value $R(p_t)$, a nonnegative quantity that may depend on the underlying price p_t. This assumption is a reduced-form representation of asset redistribution as a consequence of strategic renegotiation between creditors and shareholders upon financial distress (e.g., Fan and Sundaresan (2000)).[6] Many cases of financial distress are resolved through debt reorganization in private workouts, with only a fraction of them actually going through bankruptcy filings.[7] In their structural estimation of a dynamic capital structure model that incorporates such debt renegotiations, Morellec et al. (2008) find that the parameter η has wide cross-sectional variation among U.S. firms with a mean of around 20% of firm value at the time of distress.

B. Equity Valuation

Under the risk-neutral measure, $\boldsymbol{Q}$, the evolution of the log price, p_t, is

$$dp_t = (\mu^{\boldsymbol{Q}} - \zeta p_t)\,dt + \sigma dW_t^{\boldsymbol{Q}}, \tag{2}$$

[6] In the structural model of Fan and Sundaresan (2000), η is the product of shareholder bargaining power and liquidation costs, both taken to be deterministic quantities. Although it is possible to consider the case of a stochastic η, adding this layer of complexity does not alter the basic intuition.

[7] See, for example, Gilson, John, and Lang (1990) and Franks and Torous (1994). Hotchkiss et al. (2008) provide an excellent review of the recent literature. In the case of bankruptcy filings, deviations from the absolute priority rule have been documented by Franks and Torous (1989), Eberhart, Moore and Roenfeldt (1990), Weiss (1991), and Betker (1995).

where $\mu^{\boldsymbol{Q}}$ is the risk-adjusted drift, and

$$dW_t^{\boldsymbol{Q}} = \gamma dt + dW_t^{\boldsymbol{P}} \tag{3}$$

is a Brownian motion under the measure $\boldsymbol{Q}$ with γ being the market price of risk associated with the price process dp_t.[8] Denoting by $E^{\boldsymbol{Q}}$ the expectation under the risk-neutral measure $\boldsymbol{Q}$, the firm's equity value is given by

$$V(p_t) = E^{\boldsymbol{Q}}\left[\int_0^{\tau_L} e^{-rs}(e^{p_{t+s}} - c - l)ds + \eta R(\underline{p})e^{-r\tau_L}\right], \tag{4}$$

where $\tau_L = \inf\{t : p_t = \underline{p}\}$ denotes the first time log price p_t hits the threshold $\underline{p}$, at which point the firm becomes distressed. For expositional convenience, we will use the terms financial distress and default interchangeably. The threshold $\underline{p}$ is chosen optimally by shareholders.[9] The integrand in equation (4) represents the stream of profits received by equity holders until default. The last term represents the present value of shareholder recovery upon default, which is a fraction η of the residual value $R(\underline{p})$. The following proposition characterizes the equity value and the endogenous default boundary.

PROPOSITION 1: *Assume that the log price evolves according to the OU process (1). Then the equity value (4) is given by*

$$V(p_t) = \begin{cases} V^U(p_t) - \dfrac{l+c}{r} + A \cdot H\left(-\dfrac{r}{\zeta}, -\dfrac{\mu^{\boldsymbol{Q}} - \zeta p_t}{\sigma\sqrt{\zeta}}\right), & \text{if } p_t > \underline{p} \\ \eta R(p_t), & \text{if } p_t \leq \underline{p} \end{cases}, \tag{5}$$

where

$$V^U(p_t) = \int_0^\infty e^{-r\tau}\left[\exp\left(p_t e^{-\zeta\tau} + \left(1 - e^{-\zeta\tau}\right)\frac{\mu^{\boldsymbol{Q}}}{\zeta} + \sigma^2\left(\frac{1 - e^{-2\zeta\tau}}{4\zeta}\right)\right)\right]d\tau, \tag{6}$$

$H(v, z)$ is the generalized Hermite function of order v, given in (A4), and the quantities A and $\underline{p}$ are constants that are determined by the following value-matching and smooth-pasting conditions:

$$V(\underline{p}) = \eta R(\underline{p}) \tag{7}$$

$$V'(\underline{p}) = \eta R'(\underline{p}). \tag{8}$$

[8] This is equivalent to assuming the existence of the following pricing kernel M_t:

$$\frac{dM_t}{M_t} = -rdt - \gamma dW_t^{\boldsymbol{P}},$$

where $\gamma = \dfrac{\mu^{\boldsymbol{P}} - \mu^{\boldsymbol{Q}}}{\sigma}$ and r is the instantaneous risk-free rate.

[9] The endogenous choice of default boundary by shareholders is a common feature in theoretical models (see, e.g., Black and Cox (1976) and Leland (1994)). Empirically, Brown, Ciochetti, and Riddiough (2006) show that default decisions are endogenous responses to anticipated restructuring outcomes.

The equity value in Proposition 1 has an intuitive structure. Before default, $p_t \geq \underline{p}$, equity value is equal to the present value of the unlevered risky cash flow, $V^U(p_t)$, minus the capitalized value of the liabilities, $(c + l)/r$, plus the present value of A units of the limited liability option $H(-r/\zeta, -(\mu^{\boldsymbol{Q}} - \zeta p_t)/\sigma\sqrt{\zeta})$. The default trigger $\underline{p}$ is optimally chosen by shareholders who anticipate the potential recovery $\eta\overline{R}(p_t)$ when p_t falls below $\underline{p}$.

It appears that, in the expression of equity value (5), financial leverage l does not have a substantially distinct role from operating leverage c. This observational equivalence between the two forms of leverage stems from the exogenous nature of both c and l in this simple model and is resolved in the general model analyzed in the Internet Appendix. However, it is important to point out that, even with exogenous operating and financial leverages, financial leverage serves an entirely different *contractual* role from operating leverage. The contractual obligation of shareholders to bondholders is binding, and the outcome of the strategic interaction between them crucially determines the potential payoff to shareholders upon financial distress. In the absence of financial leverage, there is no renegotiation as equity holders own all claims.

The expression for the equity value in Proposition 1 simplifies considerably in the limit as the mean reversion parameter ζ vanishes, that is, the price p_t evolves according to a Brownian motion. In order to obtain a fully closed-form solution, in the next corollary we assume that the residual firm value $R(p_t)$ is affine in the product price e^{p_t}, that is, $R(p_t) = a + be^{p_t}$, $a, b > 0$. This choice includes situations in which, upon the resolution of financial distress, equity holders receive either a fixed payout ($b = 0$) or a stake in the unlevered firm ($a = 0$, $b = 1/\delta$, as in Fan and Sundaresan (2000)). The choice, however, does not affect the underlying intuition, as we will discuss below.

COROLLARY 1.1: *Suppose the mean reversion parameter $\zeta \to 0$ in (1). Let $\delta \equiv r - \mu^{\boldsymbol{Q}} - \frac{\sigma^2}{2} > 0$, and assume that the residual firm value upon default is $R(p_t) = a + be^{p_t}$ with $a \geq 0$ and $0 \leq b \leq \frac{1}{\eta\delta}$. Then the equity value (4) is given by*

$$V(p_t) = \begin{cases} \dfrac{e^{p_t}}{\delta} - \dfrac{c+l}{r} + Ae^{\phi\, p_t}, & \text{if } p_t > \underline{p} \\ \eta(a + be^{p_t}), & \text{if } p_t \leq \underline{p} \end{cases}, \tag{9}$$

where

$$\phi = \frac{1}{2} - \frac{2(r-\delta) + \sqrt{\left(\sigma^2 - 2(r-\delta)\right)^2 + 8\sigma^2 r}}{2\sigma^2} < 0 \tag{10}$$

and the constants $\underline{p}$ and A are, respectively,

$$\underline{p} = \log\left(\frac{\eta a + \dfrac{c+l}{r}}{\left(\dfrac{1}{\delta} - \eta b\right)\left(1 - \dfrac{1}{\phi}\right)}\right) > 0 \quad \text{and} \quad A = \frac{1}{\phi}\left(\eta b - \frac{1}{\delta}\right) e^{\underline{p}(1-\phi)} > 0. \tag{11}$$

It is easily shown that the distress threshold, $\underline{p}$, is increasing in η. This is consistent with the finding in Bharath, Panchapegesan, and Werner (2009) that, in recent years, shareholder recovery in Chapter 11 proceedings is much lower and hence firms filing for bankruptcy tend to be in much worse financial condition.

The condition $b \leqslant 1/(\eta\delta)$ in the above corollary ensures that the number of limited liability put options is nonnegative ($A \geq 0$ in (11)) and that the default threshold $\underline{p}$ is well defined. Substituting the expression of A in (9) we obtain, for $p_t \geq \underline{p}$,

$$V(p_t) = \frac{e^{p_t}}{\delta} - \frac{c+l}{r} + \pi_t \left[\frac{e^{\underline{p}}}{\phi} \left(\eta b - \frac{1}{\delta} \right) \right] > 0, \tag{12}$$

where

$$\pi_t = e^{\phi(p_t - \underline{p})} \in [0, 1] \tag{13}$$

is the risk-neutral probability of default. The quantity in square brackets in equation (12) can be thought of as the payoff from the limited liability option when it expires in the money with probability π_t.

Due to the availability of a closed-form solution, the geometric Brownian motion case is useful for studying the main mechanism linking financial distress and properties of equity return. In what follows, we rely on the equity value in Corollary 1.1 to derive analytical relationships between default probability and equity return characteristics, including equity beta and return autocorrelation. We then use Proposition 1 to verify numerically that such relationships are robust to different specifications of the stochastic process describing the evolution of product prices.

C. Equity Beta, Expected Returns, and Default Probability

Our main focus in this simple framework is to examine the effects of leverage and default probability on a firm's expected return. In the model, the product price p_t is the only state variable. Following a standard argument, we measure the risk of equity with respect to p_t as

$$\beta_t = \frac{d \log V(p_t)}{dp_t}. \tag{14}$$

Hence, the (instantaneous) expected return on equity is given by

$$ER_t = r + \beta_t \lambda, \tag{15}$$

where $\lambda = \mu^{\boldsymbol{P}} - \mu^{\boldsymbol{Q}}$ denotes the risk premium associated with the price process p_t. Note that β_t in expression (15) is *not* the CAPM beta, and this stylized model is silent about the systematic risk structure of the product price process. For ease of exposition, we nevertheless refer to the quantity in (14) as the "equity beta" because, in our setting, this is the only determinant of equity risk.[10]

Using the expression for equity value derived in Proposition 1, we can compute equity beta from (14). Because its default threshold is not available in closed form, the mean reversion case does not lend itself to further analytic characterization of equity beta. For the geometric Brownian motion case of Corollary 1.1, however, we can obtain a decomposition of equity beta that highlights the interaction between a firm's book-to-market ratio and its default probability, as described in the following corollary.

COROLLARY 1.2: *Assume that the log price process p_t follows a Brownian motion, and that the firm's residual value $R(p_t)$ upon default is as specified in Corollary 1.1. Then the levered equity beta can be expressed as*

$$\beta_t = 1 + \underbrace{\left(\frac{(c-l)/r}{V(p_t)}\right)}_{BE/ME}\underbrace{\left(\frac{c+l}{c-l}\right)\left(1-\pi_t\frac{\eta ar + c + l}{c+l}\right)}_{Distress}. \tag{16}$$

The firm's revenue beta is normalized to one. The term labeled "BE/ME" represents the equity book-to-market ratio. Because of the lack of an explicit account for capital in this simple model, we take the capitalized value of fixed cost, c/r, as a proxy for the book value of assets, following Carlson et al. (2004). Similarly, we use the capitalized value of coupons, l/r, as a proxy for the book value of debt. The quantity $(c-l)/rV(p_t)$ can hence be interpreted as a proxy for the equity book-to-market ratio. In the general model of the Internet Appendix, we explicitly account for installed capital and obtain a measure of the book-to-market ratio similar to that used in empirical work.

The term labeled "Distress" in (16) captures the impact of financial leverage and distress on equity beta. Financial leverage directly affects equity beta through the limited liability provision. This is reflected in the negative sign appearing in front of the default probability π_t in (16). This negative sign might suggest that equity risk is always declining with default probability. This argument, however, is not accurate because it neglects the indirect effect of financial leverage on β_t through equity value and default probability.

More important, the effect of financial leverage depends crucially on the magnitude of shareholder recovery, as captured by the parameter η. In the absence of shareholder recovery, that is, when $\eta = 0$, it is possible to show that as the firm approaches default ($\pi_t \to 1$), the equity value $V(p_t)$ approaches zero at a faster rate than the quantity $1 - \pi_t$, causing β_t to increase with default probability and explode to infinity as π_t tends to one. In contrast, in the

[10] We note that there is a one-to-one correspondence between the (conditional) CAPM beta and our equity beta measure (β_t), which are linked through the covariance of the p_t process and the pricing kernel in the economy. The expected return on equity may thus be further expressed as $ER_t = r + \beta_t \cdot SR \cdot \rho \cdot \sigma$, where SR is the maximal Sharpe ratio attainable in the economy, and $-\rho$ is the correlation of the price process p_t with the pricing kernel in the economy. This implies that the risk premium λ associated with the output price p_t is $\lambda = SR \cdot \rho \cdot \sigma$.

presence of shareholder recovery, that is, when $\eta > 0$, the equity value $V(p_t)$ is bounded away from zero as $\pi_t \to 1$. If the beta of the residual firm value is finite, and equity holders receive a fraction of this residual value upon the resolution of distress, equity will increasingly become less risky as the default boundary is approached. This implies that, for sufficiently high levels of default probability, equity risk declines with π_t. The following corollary formally proves this intuition for the case in which the underlying price follows a geometric Brownian motion.

COROLLARY 1.3: *Assume that the log price process p_t follows a Brownian motion and the residual firm value upon default is specified as in Corollary 1.1. Then,*

(i) *If $\eta = 0$, equity beta and expected returns are monotonically increasing in default probability π_t, with $\lim_{\pi_t \to 1} \beta_t = \infty$.*
(ii) *If $\eta > 0$, equity beta and expected returns are increasing in default probability when π_t is small and decreasing in default probability when $\pi_t \to 1$.*

Corollary 1.3 provides a characterization of equity beta that is valid when the underlying price process follows a geometric Brownian motion. To assess the robustness of this result throughout the entire spectrum of default probabilities and for the case in which the underlying price process is mean reverting, we resort to numerical analysis.

In Figure 1, we report the relationship between equity beta and default probability. Panel A presents the case in which there is no shareholder recovery upon financial distress ($\eta = 0$), whereas Panel B presents the case in which shareholders are capable of recovering 2% of the asset value upon distress. These graphs serve the purpose of demonstrating qualitative patterns that are robust to a wide range of parameter choices. In both panels, the left graph refers to the case in which the log price p_t follows a Brownian motion whereas the right graph refers to the case with a mean-reverting OU process. Each graph is obtained by choosing different price levels $p_t \geq \underline{p}$ and recording the corresponding values for beta and 1-year-ahead default probability.[11] For our purposes, varying p_t is qualitatively equivalent to considering a cross-section of firms with different characteristics (leverage, operating cost, scale, etc.). The ultimate goal is to produce a cross-section of "distances to default" against which we match the corresponding betas.

We choose the salvage value $R(p_t)$ to be the book value of assets c/r, that is, $a = c/r$ and $b = 0$ in the characterization of $R(p_t)$ in Corollary 1.1. This choice

[11] Although for our theoretical derivations we refer to π_t as the "probability of default," in our numerical analysis we adhere to industry practice and calculate the default probability under the physical measure. For the geometric Brownian motion, the default probability under the real measure $\boldsymbol{P}$ is available in closed form and is provided in Lemma 1 of the Appendix (equation (A5)). For the mean-reverting case, we discretize the OU process using Tauchen's quadrature (see Tauchen (1986)) with 100 grid points and numerically compute the T-period-ahead default probability. Notice, however, that the use of the risk-neutral probability of default π_t does not alter any of the properties we derive in this section because the two quantities are monotonically related.

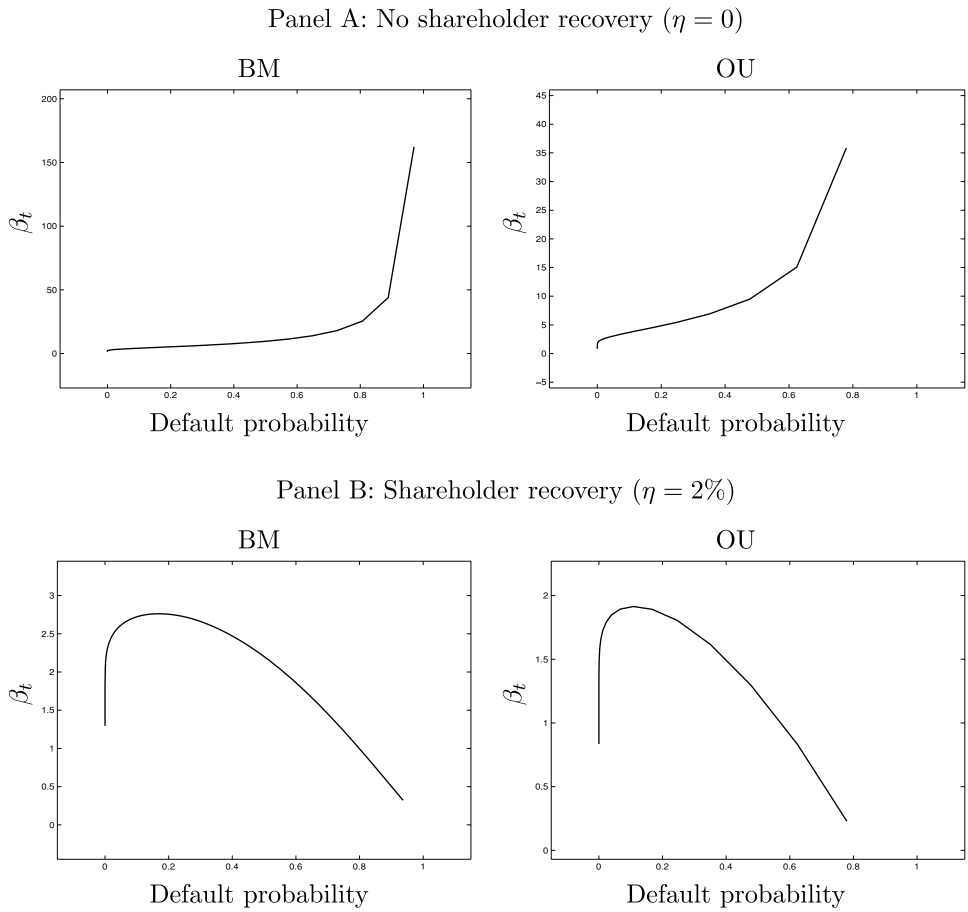

Figure 1. Equity beta and default probability. The figure reports the equity β as a function of 1-year-ahead default probability. The probability is computed according to equation (A5) in Lemma 1 of the Appendix for the Brownian motion (BM), and is obtained numerically for the Ornstein–Uhlenbeck (OU) case using Tauchen (1986) quadrature to discretize the process for p_t with 100 grid points. The parameters used for the graphs are: $\sigma = 0.3$, $\mu^{\boldsymbol{P}} = -\sigma^2/4$ (to normalize the long-run expected revenues from a unit of production to one), $r = 0.02$, $\zeta = 0.1$ (OU), $\zeta = 0$ (BM), $\lambda = 0.05$, and $c = 1$. Panel A refers to the case of no expected shareholder recovery upon financial distress, $\eta = 0$, whereas Panel B refers to the case in which $\eta = 2\%$.

allows us to measure shareholder recovery as a fraction η of the book value of assets, a variable frequently used in empirical studies (see, e.g., Eberhart et al. (1990)). As we will argue below, imposing a constant salvage value does not affect the qualitative nature of our results.

Panel A of Figure 1 shows that when there is no possibility for shareholder recovery, that is, $\eta = 0$, equity beta increases monotonically with default probability. As the default boundary is approached, equity beta explodes and the equity value goes to zero, as stated in Corollary 1.3. In contrast, Panel B shows that when expected shareholder recovery is set at a modest level of $\eta = 2\%$

of the book asset value, equity beta (and hence expected returns) exhibits a hump shape with respect to default probability. This finding is consistent with the empirical evidence in Dichev (1998), who documents a distinct hump in the stock return relationship with accounting-based measures of distress (Altman's Z and Ohlson's O scores), and in Campbell et al. (2008), who illustrate hump-shaped factor loadings with respect to default likelihood.[12]

The intuition for the hump shapes in the graphs is as follows. As financial leverage amplifies the level of equity risk (relative to asset risk), at low levels of distress likelihood equity beta increases with leverage and hence with default probability. At high levels of distress likelihood, however, the prospect of recovering a fraction of assets, which have a lower beta than the levered equity, increasingly counterweighs the amplifying effect of leverage in determining the risk of equity. As the firm inches closer to the point of distress, the likelihood of recovery "mutates" the risk of levered equity into the risk of the underlying, safer, asset $R(p_t)$. When $R(p_t)$ is modeled as the (constant) book value of assets, equity beta is, in fact, converging to zero at high levels of default probability, as shown in Panel B. Note, however, that when $\eta > 0$, the relation between beta and default probability is bound to be hump shaped, regardless of the form of the residual value $R(p_t)$, as long as this quantity is positive and has finite beta.

The hump shape in the relationship between levered equity beta and default probability, a consequence of shareholder renegotiation power in the event of financial distress, has interesting implications for the cross-sectional properties of equity returns. In the next two subsections, we elaborate on how such a relationship affects two widely studied cross-sectional anomalies: the value premium and momentum in stock returns.

D. Value Premium and Financial Distress

The decomposition of equity beta in Corollary 1.2 illustrates that the cross-sectional variation in betas is attributable to the interaction between the "book-to-market" and financial distress effects. The way in which beta depends on default probability has implications for the relationship between default probability and the value premium, that is, the return spread between stocks with high vs. low book-to-market ratios. We claim that if beta is monotonically increasing and convex in default probability, as in Panel A of Figure 1, then the value premium is also positive and increasing. In contrast, if beta is hump shaped in default probability, as in Panel B, then the value premium is positive for low levels of default probability and negative otherwise.

The intuition for the predicted patterns of the value premium at different levels of default probability is as follows. Suppose we have two firms with identical book values, default probabilities, and stock prices. One of them experiences a negative shock to its stock price, whereas the other experiences a positive

[12] The empirical evidence for the hump may have a different appearance from that of Figure 1 because, as most stocks cluster at low levels of default probability, portfolio sorting procedures usually tend to stretch out the hump shape to higher levels of default probability.

shock and hence has a higher stock price. The first stock will have a larger book-to-market ratio and a higher default probability than the second stock. If both stocks have $\eta = 0$, then from Panel A of Figure 1, the difference between their expected returns, that is, the value spread, should be positive and upward sloping with respect to default probability. In contrast, if both firms have $\eta > 0$, then Panel B of Figure 1 indicates that the return spread between high book-to-market stocks and low book-to-market stocks should be positive for low levels of default probability and negative for high levels.

Empirically, the value premium is computed by forming portfolios of stocks with different book-to-market ratios. Given the cross-sectional heterogeneity in the degree of shareholder recovery, η, the above argument allows us to conjecture, and numerically confirm, that value spreads should be humped with respect to default probability, that is, upward sloping at low levels of default probability and downward sloping at high levels. We verify that this intuition is robust in the general model solved in the Internet Appendix and supported by the data in the empirical analysis of Section II.

E. Momentum and Financial Distress

The relation between equity beta and default probability is also important for understanding return autocorrelation and the properties of momentum strategies.

Equity returns exhibit a positive return autocorrelation if the expected return increases with realized returns. Intuitively, return autocorrelation can be thought of as the slope obtained by regressing the instantaneous change in expected returns on realized returns. Assuming that the underlying state variable p_t follows the stochastic process in (1), and using Itô's lemma and the definition of β_t given in (14), the covariance between changes in expected returns and realized returns is given by $\lambda\sigma d\beta_t/dp_t \times \sigma\beta_t$, and the variance of realized returns is $\sigma^2\beta_t^2$. Combining these two quantities, we obtain the following expression for autocorrelation in returns:

$$AC(p_t) = \frac{\lambda}{\beta_t}\frac{d\beta_t}{dp_t} = \frac{\lambda}{\beta_t}\frac{d^2\log(V(p_t))}{dp_t^2}, \tag{17}$$

where the last equality follows from the definition of β_t. A positive return autocorrelation on the stock level is a sufficient, though not necessary, condition for the profitability of momentum strategies. Hence, strategies based on portfolios of stocks with positive autocorrelations should result in enhanced momentum profits.

For the case in which the underlying price process follows a geometric Brownian motion, we can explicitly derive an expression for the autocorrelation coefficient and analyze its relationship with default probability, as stated in the following corollary.

COROLLARY 1.4: *Suppose the log price p_t follows a Brownian motion and the residual firm value upon default is specified as in Corollary 1.1. Then the* *autocorrelation in equity returns is given by*

$$AC(p_t) = \lambda \left[1 - \beta_t - \pi_t \frac{\phi(\eta ar + c + l)}{r\beta_t V(p_t)} \right]. \tag{18}$$

If $\eta = 0$, then $AC(p_t) < 0$. If $\eta > 0$, then there exists a $p^*(\eta)$ such that $AC(p^*(\eta)) = 0$. For all $\underline{p} \le p_t < p^*(\eta)$, $AC(p_t) > 0$, and for all $p_t > p^*(\eta)$, $AC(p_t) < 0$. Furthermore, $p^*(\eta)$ is increasing in η.

This corollary highlights the crucial role of financial distress and the ensuing potential recovery for equity holders in the determination of stock return continuation. The corollary states that return autocorrelation is positive only in the presence of shareholder recovery $\eta > 0$ and for sufficiently high levels of default probability, that is, $p_t < p^*(\eta)$. The intuition behind this result stems from the humped relationship between expected returns and default probability discussed above. Because beta is a hump-shaped function of default probability, as the firm edges toward default with a declining stock price, the ex-ante level of equity risk decreases. Similarly, as the firm moves away from the brink of default, its stock price and equity risk rise. Both scenarios depict a return pattern that exhibits a positive autocorrelation. Because this mechanism applies *only* to firms with high default probability and $\eta > 0$, the risk dynamic we highlight here is consistent with the recent empirical finding of Avramov et al. (2007) that the momentum effect in stock returns is driven primarily by firms with low credit ratings.

There are three points worth noting. First, our model is capable of endogenously generating positive return autocorrelations among firms with low credit quality and high expected shareholder recovery upon default even when the underlying product price process is not predictable, as is the case when p_t follows a Brownian motion. Our result stems uniquely from the existence of a hump-shaped relationship between beta and default probability, which is also present in the case of a mean-reverting log price process, as shown in Panel B of Figure 1. Therefore, the mechanism described in Corollary 1.4 extends naturally beyond the Brownian motion case.

Second, as Johnson (2002) and Sagi and Seasholes (2007) point out, autocorrelation in equity returns is positive if the log equity value is convex in the log price p_t, that is, $d^2 \log(V(p_t))/dp_t^2 > 0$, as long as the risk premium λ and β_t are positive. In our model, log convexity obtains because of the presence of positive shareholder recovery when $\eta > 0$ and the output price $p_t < p^*(\eta)$, which implies that the default probability π_t is sufficiently high. Figure 2 illustrates this point by plotting the log equity value in the absence of shareholder recovery (Panel A) and with positive shareholder recovery (Panel B). As demonstrated in the figure, a positive value of η dramatically changes the curvature of the log equity value, compared to the case of no shareholder recovery. In Panel B, where $\eta = 2\%$, the log equity value is convex for low levels of the log price p_t and concave elsewhere. Low levels of log price are states with high default probabilities. Shareholder recovery introduces log convexity in equity values

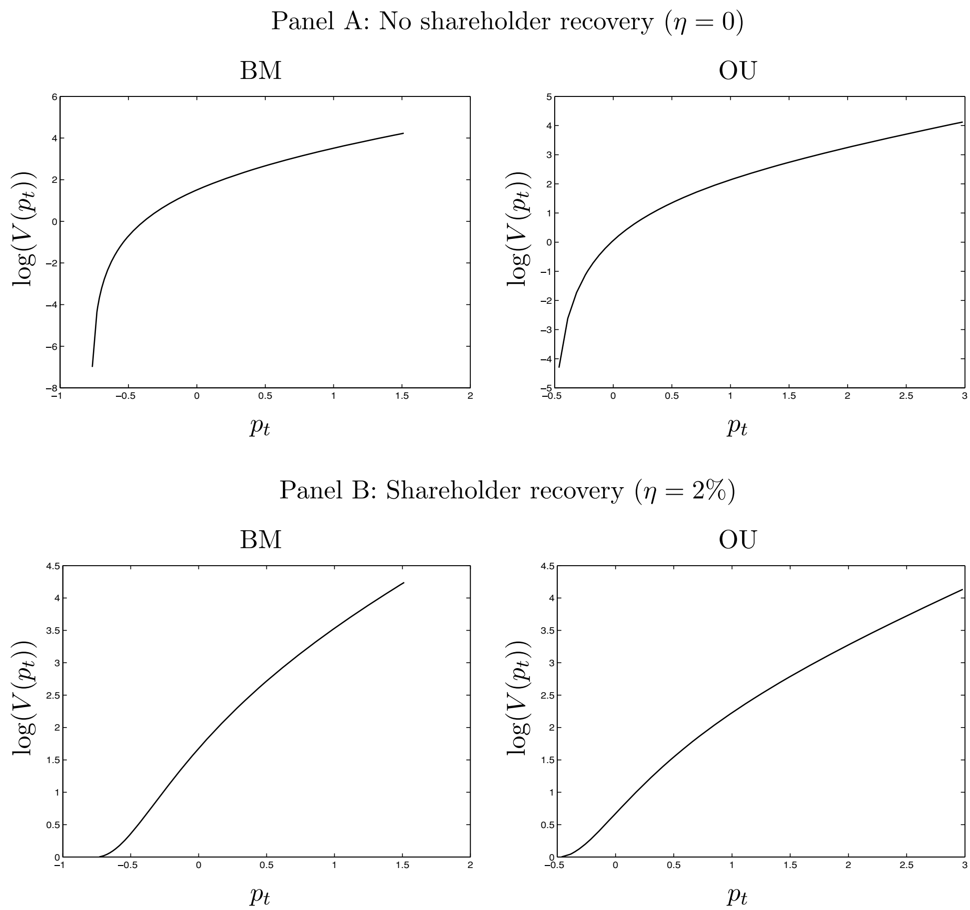

Figure 2. Shareholder recovery and log convexity. The figure reports the logarithm of equity value, $\log(V(p_t))$, for the Brownian motion (BM) and Ornstein–Uhlenbeck (OU) cases of Proposition 1 as a function of the log price p_t. Parameter values are the same as in Figure 1, and shareholder recovery value is set to be a fraction η of the book value of assets, c/r.

when default probability is high, hence generating positive autocorrelation. In contrast, in Sagi and Seasholes (2007), growth options and the absence of financial leverage are instrumental for inducing log convexity in equity values, and hence positive return autocorrelation. Therefore, the two mechanisms for generating enhanced momentum profits are complementary.

Finally, the fact that $p^*(\eta)$ is increasing in η implies that, when shareholder recovery is high, positive autocorrelation persists over a larger range of log prices p_t. This property has implications for both the persistence and the strength of momentum profits and leads to a novel prediction on the cross-sectional variation of enhanced momentum strategies. Because, for nearly distressed stocks, the persistence of their positive return autocorrelation increases with the prospect of shareholder recovery upon distress, momentum profits can be enhanced by concentrating on stocks with strong prospects of shareholder recovery. This prediction is shown to be robust in the general model solved in the Internet Appendix and confirmed in the empirical analysis of Section II.

F. Discussion

The simple model discussed in this section yields insights into several puzzling pieces of empirical evidence. First, because of the interaction between leverage and book-to-market in the determination of beta, for most firms—with the exception of low credit quality firms for which shareholders expect a nonzero recovery value in distress renegotiations—the risk of assets-in-place to equity holders is amplified by financial leverage, implying that the magnitude of the book-to-market effect is stronger for more heavily levered firms. This is consistent with the evidence that the value premium is most significant for firms with high default probability (see, e.g., Griffin and Lemmon (2002), Vassalou and Xing (2004), and Chen (2009)).

Second, our model is useful for understanding the results of Hecht (2004) and Choi (2009) that firm-level asset returns do not exhibit strong cross-sectional patterns, such as the book-to-market and momentum effects. As our model shows, these patterns are generally enhanced by leverage, and their magnitude in asset returns may be too small to be statistically and economically significant. The impact of financial leverage on cross-sectional returns is also suggested by Ferguson and Shockley (2003), who argue that the SMB and HML factors in the Fama–French three-factor model are instruments for measurement error in equity beta because of the absence of debt in the proxy market portfolio. However, this argument ignores the time-varying nature of beta as well as its dependence on firms' characteristics, as highlighted in our framework. Moreover, the nonlinearity in the equity payoff introduced by leverage and the limited liability option in our model helps provide a plausible justification for the relationship between conditional skewness and stock returns documented in Harvey and Siddique (2000).

More important, our simple model shows that accounting for potential shareholder recovery upon financial distress produces a rich set of implications for cross-sectional properties of stock returns. The resulting hump-shaped relationship between expected returns and default probability leads to a testable new prediction of a humped value premium with respect to default probability. It also provides an explanation for the recently documented evidence on the concentration of momentum profits in low credit quality firms and further predicts that momentum profits will be stronger for nearly distressed firms with higher expectations of shareholder recovery.

II. Empirical Analysis

In this section, we first provide direct evidence of the hump-shaped relationship between a firm's conditional beta and default probability. We then empirically verify the novel predictions of our model regarding the cross-sectional properties of the value premium and momentum profits.

A. Data

To gauge a firm's default probability, we use a market-based measure—EDF—obtained directly from MKMV. The data are available at a monthly frequency. A firm's EDF measure represents an assessment of the likelihood of default for that firm within a year. This measure is constructed from market-traded stock prices based on the Vasicek–Kealhofer model (Kealhofer (2003a,b)), which adapts the Black–Scholes (1973) and Merton (1974) framework and is calibrated through a comprehensive database of historical default experiences.[13]

We match the EDF database with the CRSP (Center for Research in Security Prices) and COMPUSTAT databases, that is, a stock needs to have data in all three databases to be included in our analysis. Specifically, for a given month, we require a firm to have an EDF measure and an implied asset value in the MKMV data set; stock price, shares outstanding, and return data from CRSP; and accounting numbers from COMPUSTAT for firm-level characteristics. We limit our sample to nonfinancial U.S. firms.[14] We also drop from our sample stocks with a negative book-to-market ratio. Our baseline sample contains 1,615,664 firm–month observations and spans from February 1969 to November 2007.[15]

Summary statistics for the EDF measure are reported in Table I. The average EDF measure in our sample is 3.30% with a median of 1.07%. Panel A shows that 75% of firms have a default probability of less than 3.5%. One should note that MKMV winsorizes EDF scores at 20%. Around 5% of the firms are assigned an EDF score of 20% at any given time.

Because many empirical studies exclude stocks with a per-share price lower than \$5 out of concern for liquidity and market microstructure issues, we examine separately distributions of the EDF measure in the subsample with a minimum per-share price of \$5 and in the subsample containing only stocks with a per-share price lower than \$5. Panel A of Table I shows that low-priced stocks tend to have much higher default probabilities, with a mean EDF of 6.89%, and more than 50% of these stocks have a higher than 4% chance of defaulting within 1 year. For high-priced stocks, default probabilities are generally low with a mean EDF of 1.15%, and 90% of these stocks have a less than 2.84% probability of defaulting within 1 year. This implies that low-priced stocks on average have greater risk of financial distress.

In our empirical examination below, for each month, we group stocks evenly into 10 deciles according to their EDF values. To better understand the property of each group, Panel B of Table I presents the time-series average of the mean

[13] See Crosbie and Bohn (2003) for details on how MKMV implements the Vasicek–Kealhofer model to construct the EDF measure.

[14] Financial firms are identified as firms whose Standard Industrial Classification (SIC) code is between 6000 and 6999.

[15] We follow Shumway (1997) and Shumway and Warther (1999) to deal with the problem of delisted firms. Specifically, whenever available, we use the delisting return reported in the CRSP data file for stocks that are delisted in a particular month. If the delisting return is missing but the CRSP data file reports a performance-related delisting code, then we impute a delisted return of either −30% (NYSE and Amex stocks) or −55% (NASDAQ stocks) in the delisting month.

Table I

Summary Statistics of the EDF Measure

Panel A reports summary statistics for the entire sample and for the subsamples of stocks with a per-share price no less than \$5 and those with a per-share price smaller than \$5. Panel B reports summary statistics of the EDF measure and of the Amihud (2001) illiquidity measure across EDF-sorted portfolios. EDF quantities are expressed in percentage. "Average EDF" refers to the time-series mean of average EDF measures within each EDF decile and "Max EDF" is the time series mean of maximum EDF measures within each EDF decile.

Panel A: EDF statistics

	Mean	Std	Min	p5	p10	p25	Median	p75	p90	Max
Full sample	3.30	5.19	0.02	0.05	0.09	0.30	1.07	3.47	10.44	20.00
Stocks with $p \geq \$5$	1.15	1.86	0.02	0.03	0.06	0.17	0.52	1.36	2.84	20.00
Stocks with $p < \$5$	6.89	6.75	0.02	0.26	0.54	1.52	4.10	10.68	20.00	20.00

Panel B: Summary statistics of EDF-sorted decile portfolios

	Low EDF	2	3	4	5	6	7	8	9	High EDF
					Full Sample					
Average EDF	0.07	0.20	0.38	0.61	0.95	1.46	2.26	3.75	6.98	14.79
Max EDF	0.13	0.28	0.48	0.76	1.17	1.79	2.84	4.93	9.93	19.82
Average Amihud measure	0.01	0.03	0.05	0.10	0.18	0.33	0.64	1.33	3.36	11.47
					Stocks with price $\geq$\$5					
Average EDF	0.06	0.14	0.23	0.36	0.52	0.73	1.03	1.47	2.24	5.21
Max EDF	0.10	0.18	0.29	0.43	0.61	0.86	1.21	1.77	2.86	19.20
Average Amihud measure	0.01	0.02	0.03	0.05	0.07	0.11	0.15	0.23	0.34	0.58

and maximum EDF measures in each EDF decile for both the full sample and the subsample with a minimum per-share price of \$5. In addition, we report the Amihud (2002) measure of illiquidity for each decile portfolio to examine the liquidity difference across decile portfolios. From the values reported in Panel B, it is evident that the biggest difference between the full sample and the subsample of stocks with a per-share price of \$5 or higher occurs in the portfolios with high levels of default probability. For example, the average EDF in the top EDF decile is 14.79% for the full sample and only 5.21% for the subsample of stocks with high per-share prices. Similarly, the Amihud illiquidity measure is 11.47 in the tenth decile for the full sample and only 0.58 in the tenth decile of the subsample. Finally, the mean values of the maximum EDF of decile portfolios indicate that the cutoff EDF values for deciles are not evenly spaced, especially for stocks with higher per-share prices.

B. Beta and Default Probability

Before analyzing how value premium and momentum vary with default probability, we provide direct empirical evidence in support of a hump-shaped relationship between conditional beta and default probability.

The calculation of conditional beta at the firm level is a notoriously thorny econometric problem. In our analysis, we follow the literature (e.g., Lewellen and Nagel (2006), Ang and Kristensen (2010), and Boguth et al. (2010)) and employ two approaches to estimate monthly equity beta using daily returns in each month from the CRSP Daily Stock File. First, we use a standard market model to estimate beta as the slope coefficient in the time-series regression $R_{i,t} = \alpha_i + \beta_i R_{M,t} + \epsilon_{i,t}$, where $R_{i,t}$ is the excess return on the stock of firm i and $R_{M,t}$ is the excess return on the value-weighted CRSP index. Second, following Dimson (1979), we augment the procedure above by including current, one-period lag, and one-period lead market returns in the regression and estimate the equity beta for each firm–month as the sum of the slopes on all three market returns.[16]

For each month, we sort stocks into deciles according to their MKMV EDF measure and, for each beta measure, we compute the value- and equal-weighted portfolio beta at the time of portfolio formation.[17] The results are reported in Table II. As the table shows, both the market model beta and the Dimson beta show a distinct hump shape in the EDF measure. Value-weighted beta increases until the seventh EDF decile and drops sharply afterward. The difference between the beta in the tenth EDF decile and that in the first EDF decile is -0.06 with a t-statistic of -2.16. Equal-weighted beta also exhibits a distinct hump shape. The difference between top and bottom EDF deciles is -0.22 (t-statistic -11.30).[18]

[16] We calculate excess returns using the 1-month T-bill rate obtained from Ken French's web page and set the estimated equity beta as missing in a month when the estimate is based on less than five observations.

[17] The results are robust if we record portfolio betas in the month following portfolio formation.

[18] The fact that equal-weighted betas are smaller than value-weighted betas is due to the noise in the beta estimation of small firms, which is known to bias beta estimates toward zero.

Table II

Conditional Equity Beta and Default Probability

Each month, stocks are sorted into deciles of MKMV's EDF score (EDF). The table reports the time-series average of value-weighted (Panel A) and equal-weighted (Panel B) conditional betas at the time of portfolio formation. Conditional betas are obtained each month from regressions using daily returns from CRSP. The *Market model beta* is obtained from the regression $R_{i,t} = \alpha_i + \beta_i R_{M,t} + \epsilon_{i,t}$, where $R_{i,t}$ and $R_{M,t}$ are, respectively, excess returns on the stock of firm i and on the value-weighted CRSP index. The *Dimson beta* is $\sum_{k=-1}^{k=+1} \beta_{i,k}$, where $\beta_{i,k}$ is obtained from the regression $R_{i,t} = \alpha_i + \sum_{k=-1}^{k=+1} \beta_{i,k} R_{M,t+k} + \epsilon_{i,t}$.

	Low EDF 1	2	3	4	5	6	7	8	9	High EDF 10	Diff.
Panel A: Value weighted											
Market model beta	1.00	1.01	1.06	1.08	1.11	1.15	1.16	1.12	1.04	0.94	−0.06
t-stat	177.07	194.08	135.17	121.21	98.64	81.75	67.93	57.42	45.39	36.09	−2.16
Dimson beta	0.94	1.03	1.09	1.11	1.16	1.20	1.21	1.17	1.09	1.03	0.09
t-stat	126.86	140.75	104.48	89.68	72.30	64.60	53.08	45.67	34.96	27.79	2.19
Panel B: Equal weighted											
Market model beta	0.77	0.79	0.81	0.81	0.80	0.79	0.77	0.72	0.64	0.54	−0.22
t-stat	115.89	101.57	87.16	77.40	70.42	61.30	52.19	44.91	36.28	26.73	−11.30
Dimson beta	0.83	0.87	0.89	0.90	0.89	0.88	0.85	0.81	0.73	0.62	−0.21
t-stat	111.04	90.15	78.78	67.84	61.43	54.29	44.42	38.17	32.72	19.10	−6.30

In summary, the results in Table II support the prediction of our theory and corroborate the indirect evidence provided in Campbell et al. (2008), Favara et al. (2010), and Garlappi et al. (2008). In the rest of this section, we carry out our main empirical investigation of the implication of this hump shape in beta for cross-sectional anomalies such as the value premium and momentum profits.

C. Value Premium and Default Probability

We first examine how the value premium changes with default probability. In each month, we sort all stocks in our full sample into 10 deciles according to their EDF scores and, independently, into three terciles according to their book-to-market ratios. We then compute both value- and equal-weighted returns of each portfolio in the month after portfolio formation and record the time-series average of the value premium, that is, the return spread between the top book-to-market portfolio and the bottom book-to-market portfolio. The results are reported in Table III.

For the full sample, the value premium initially rises with default probability and then starts to decline at high levels of default probability. For value-weighted returns, the value premium rises from 0.05% per month in the first EDF decile to 1.60% in the eighth decile and then drops to 1.07% in the last decile. This hump-shaped pattern is more pronounced with equal-weighted returns, where the value premium rises from 0.11% per month in the first EDF decile to 1.37% in the eighth decile and then drops to 0.44% in the last decile. This hump shape in the relationship between the value premium and default probability is consistent with the prediction of our theoretical model.[19]

Table III further shows that the pattern in raw returns persists in risk-adjusted returns obtained from the CAPM ("CAPM α"), the Fama and French (1993) three-factor model ("FF α"), and the momentum-augmented four-factor model of Carhart (1997) ("4-Factor α"), respectively. This pattern is present even after accounting for the Pástor and Stambaugh (2003) liquidity factor ("5-Factor α"). Moreover, in order to mitigate the liquidity effect and market microstructure issues concerning stocks with high default probabilities and/or low prices per share, we record and analyze the return patterns in the second month after portfolio formation as suggested in Da and Gao (2010). The results, reported in the Internet Appendix, are similar to those reported in Table III.

The above results appear to contradict earlier evidence in the literature that documents a larger value spread among firms with higher default probability (e.g., Griffin and Lemmon (2002)). This discrepancy is, however, illusory and has much to do with the sample selection procedure and/or with the coarseness of the sorts used to classify stocks into portfolios. A frequently used sample filtering rule is to exclude stocks with per-share price less than $5 to

[19] Recently, Choi (2010) also found that the value premium is humped with respect to the Ohlson (1980) *O*-score.

Table III

Value Premium and Default Probability

Each month, stocks are sorted independently into terciles of book-to-market ratios (B/M) and deciles of MKMV's EDF scores (EDF). The table reports the time-series average of value- and equal-weighted returns of each portfolio obtained in the month after portfolio formation. Portfolio returns are expressed in percentage per month. CAPM-alpha, FF-alpha, 4-Factor alpha, and 5-Factor alpha refer to the value premium after controlling for risk according to, respectively, the CAPM market factor, the Fama–French three-factor model, the Carhart four-factor model, and a five-factor model that also includes the liquidity factor of Pástor and Stambaugh (2003).

Low EDF		High EDF								
	1	2	3	4	5	6	7	8	9	10
Panel A: Value weighted										
Returns										
Low B/M	0.94	0.96	0.71	0.93	0.74	0.40	0.06	−0.25	−0.49	−0.19
Medium B/M	0.99	1.02	1.21	1.27	1.01	1.20	0.99	0.68	0.46	0.32
High B/M	0.99	1.11	1.23	1.47	1.27	1.42	1.35	1.34	0.92	0.88
Value Premium										
Raw	0.05	0.15	0.52	0.54	0.53	1.03	1.29	1.60	1.41	1.07
t-stat	0.275	0.745	2.334	2.246	2.199	4.111	4.747	5.437	4.468	2.989
CAPM alpha	0.15	0.34	0.74	0.78	0.74	1.24	1.46	1.75	1.60	1.17
t-stat	0.789	1.841	3.700	3.622	3.352	5.349	5.566	6.141	5.263	3.273
FF alpha	−0.44	−0.22	0.14	0.17	0.20	0.71	0.96	1.16	1.17	0.67
t-stat	−3.103	−1.673	0.983	1.068	1.081	3.531	3.981	4.509	3.945	1.933
4-Factor alpha	−0.53	−0.35	0.01	0.05	−0.02	0.58	0.74	0.84	0.73	0.33
t-stat	−3.653	−2.596	0.038	0.304	−0.134	2.805	3.038	3.261	2.507	0.935
5-Factor alpha	−0.54	−0.37	0.02	0.12	−0.08	0.53	0.68	0.90	0.72	0.28
t-stat	−3.638	−2.704	0.150	0.726	−0.428	2.538	2.757	3.486	2.445	0.785
Panel B: Equal weighted										
Returns										
Low B/M	1.03	0.96	0.78	0.78	0.58	0.51	0.39	0.16	0.28	1.71
Medium B/M	1.03	1.14	1.31	1.33	1.28	1.41	1.15	1.08	0.82	1.91
High B/M	1.13	1.19	1.43	1.50	1.48	1.55	1.62	1.53	1.48	2.14
Value Premium										
Raw	0.11	0.23	0.65	0.72	0.90	1.05	1.24	1.37	1.19	0.44
t-stat	0.651	1.206	3.469	3.517	4.605	5.012	6.110	6.271	4.703	1.498
CAPM alpha	0.28	0.47	0.88	0.96	1.12	1.28	1.45	1.56	1.38	0.60
t-stat	1.937	3.058	5.778	5.705	6.766	7.155	8.141	7.796	5.786	2.118
FF alpha	−0.19	−0.02	0.41	0.47	0.67	0.82	1.06	1.17	1.08	0.31
t-stat	−1.832	−0.248	4.138	4.172	5.452	5.873	6.883	6.546	4.676	1.101
4-Factor alpha	−0.22	−0.09	0.28	0.34	0.48	0.64	0.78	0.83	0.58	−0.07
t-stat	−2.148	−0.888	2.798	2.974	3.989	4.597	5.287	4.862	2.695	−0.244
5-Factor alpha	−0.21	−0.09	0.25	0.33	0.46	0.62	0.74	0.82	0.55	−0.10
t-stat	−1.967	−0.911	2.508	2.871	3.788	4.455	4.953	4.722	2.536	−0.339

avoid potential market microstructure issues. As illustrated in Table I, this filtering rule excludes precisely those stocks with high levels of default probability. Therefore, it is likely that the extant empirical evidence reflects the variation in the value premium over a limited range of default likelihood, in particular, the range in which the value premium increases with default probability, as indicated by our theory. Alternatively, some studies, such as Griffin and Lemmon (2002) and Vassalou and Xing (2004), sort stocks into quintile portfolios according to their default likelihood, a coarse classification that may elude finer features of the relationship between default probability and value spread.

To verify that data selection and classification can affect the results examined here, we first sort all stocks, including the low-priced ones, into quintiles (instead of deciles) according to their EDF scores and repeat the procedure for Table III. In Panel A of Figure 3, we present the value spread based on value-weighted returns of book-to-market sorted portfolios. The panel shows that with a coarse rank of default probability quintiles, we only observe a generally positive association between default likelihood and value spread, as reported in the previous studies. We next restrict our sample of stocks to those with a per-share price of \$5 or higher and repeat the same procedure of portfolio formation and return recording used in Table III. The result, presented in Panel B of Figure 3, confirms that for this subset of stocks the value premium is indeed increasing in EDF scores.

Because Table I shows a close link between high default probabilities and low stock prices, the result illustrated in Panel B of Figure 3 could be the consequence of still-low levels of default probability in the high EDF deciles among high-priced stocks. Indeed, Table I shows that the 75$^{\text{th}}$ percentile EDF score for high-priced stocks is 1.36%, compared to 10.68% for low-priced stocks and 3.47% for the full sample, respectively. If our prediction is robust, then it should also apply to high-priced stocks with high levels of default probability that are comparable to those of low-priced stocks. To examine this conjecture, we use the breakpoints for the 10 EDF portfolios based on the full sample, including low-priced stocks, but sort only stocks with a per-share price of \$5 or higher into their respective portfolios.[20] The value spread presented in Panel C of Figure 3 exhibits a striking hump shape that is absent in Panel B. This finding demonstrates the validity of our prediction and at the same time mitigates the concern about illiquidity typically associated with low-priced stocks.

D. Momentum Profits, Shareholder Recovery, and Default Probability

Our model predicts that momentum profits in stock returns are likely to be more pronounced for firms with high levels of default probability. This is consistent with the evidence in Avramov et al. (2007), who document that among stocks with S&P firm-level credit ratings, those with poor credit ratings are most important in generating momentum profits. Furthermore, our model yields a unique prediction regarding how expected shareholder recovery can affect cross-sectional patterns of momentum.

To test the prediction of our model, we first need proxies for the shareholder recovery parameter η. One component of shareholder recovery is shareholders' "bargaining power." Shareholder recovery is larger when shareholders' bargaining power is greater. To capture this dimension, we rely on two proxies used in the literature to measure shareholder bargaining power:

[20] We thank an anonymous referee for this suggestion.

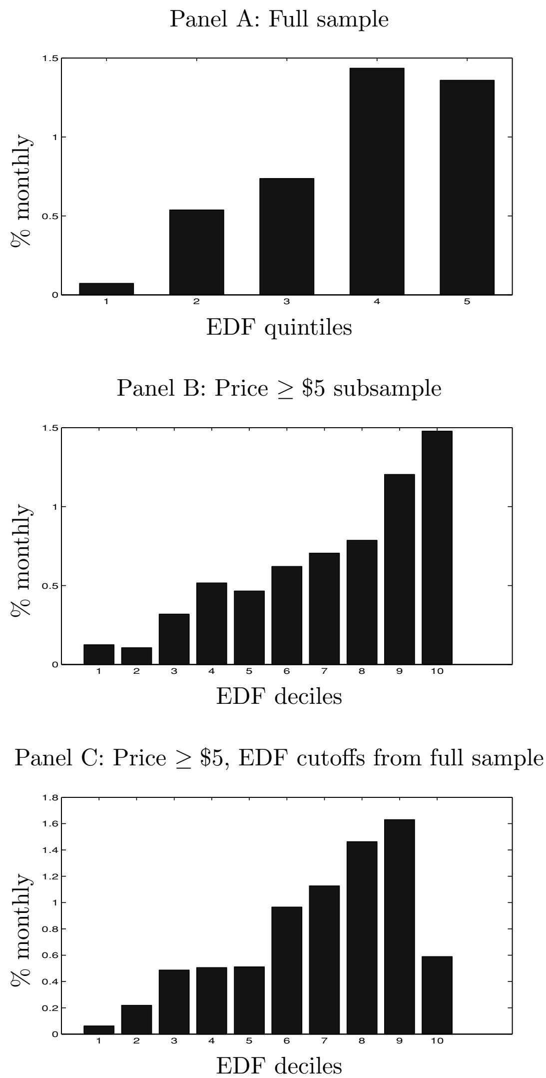

Figure 3. Value premium and default probability. The figure reports the value spread based on value-weighted returns of book-to-market sorted portfolios. Panel A considers the full sample of stocks from 1969 to 2007 and uses expected default frequency (EDF) quintiles. Panel B considers stocks with per-share prices of $5 or higher. Panel C also considers stocks with per-share prices of no less than $5 but computes the cutoffs for EDF deciles from the full sample of stocks.

(i) asset size and (ii) R&D expenditures. This choice is supported by the studies of Franks and Torous (1994) and Betker (1995), who document that deviation from the absolute priority rule is positively related to firm size, and of Opler and Titman (1994), who show that firms with high R&D costs suffer the most in financial distress and may be subject to liquidity shortage that diminishes the bargaining power of shareholders in renegotiation. In our empirical analysis, we take firms with larger asset bases or lower R&D expenditures as more likely to have a larger η.

A second component of shareholder recovery is the "liquidation cost" incurred in the event of asset liquidation. All else equal, shareholder recovery is high when liquidation costs are high because high liquidation costs provide incentives for debt holders to renegotiate debt in order to avoid the deadweight costs of liquidation, and hence increase the likelihood and the amount of shareholder recovery. To capture this aspect of the renegotiation process, we rely on the concept of asset specificity. As Shleifer and Vishny (1992) argue, when a firm's assets are specific or unique to a particular industry, they are likely to be subject to substantial fire-sale discounts in liquidation auctions. Therefore, all else being equal, liquidation costs increase when a firm's assets are more specific. We gauge a firm's asset specificity using a measure of industry concentration: the Herfindahl index of sales in an industry.[21] Firms in a more concentrated industry are likely to have more specific assets, and hence larger liquidation costs, that is, a larger η.

In summary, our empirical analysis on momentum strategies is based on three proxies for shareholder recovery: asset size, R&D expenditures, and industry concentration.[22] Based on these proxies, our theory predicts that, among firms with high levels of default probability, momentum profits should be stronger when (i) asset size is large, (ii) R&D expenditures are small, or (iii) industry sales concentration is high.

The methodology we follow to construct momentum profits is adapted from the "6-1-6" strategy in Jegadeesh and Titman (1993). At the beginning of each month t, we sort stocks independently into: (i) five quintiles based on their returns over the formation period $t-7$ to $t-2$, (ii) three terciles based on their EDF measures at time $t-2$, and (iii) three terciles based on a proxy for η at time $t-2$. Following the sort, the "winner" portfolio makes a fixed \$1 value-weighted investment in the top quintile stocks and sells stocks that were similarly added to the portfolio at the beginning of month $t-6$. The "loser" portfolio is defined similarly using the bottom quintile stocks. Momentum profits are defined as the difference between the returns of the winner and loser portfolios over the 6-month period from $t+2$ to $t+7$, after skipping a month following portfolio formation. We report monthly average returns in Table IV.

[21] The Herfindahl index on sales in industry j at time t is defined as

$$SalesHfdl_j(t) = \sum_{i=1}^{N_j(t)} s_{ij}^2(t),$$

where $s_{ij}(t)$ denotes the sales of firm i at time t as a fraction of total sales in industry j, and $N_j(t)$ denotes the number of firms in industry j at time t.

[22] Garlappi et al. (2008) also use the book-to-market ratio as a proxy for liquidation costs, as a low book-to-market ratio may imply that the firm is worth more as a going concern than the book value of its existing assets. Even though the results using this proxy are consistent with the model prediction, we refrain from using it here because of the multiple roles the book-to-market ratio plays, which may confound its interpretation in this context.

Table IV

Momentum Profits, Default Probability, and Shareholder Recovery

Each month, all stocks are sorted independently into terciles of EDF scores, terciles of a proxy for expected shareholder recovery, and quintiles of winners/losers according to past 6-month returns. We skip a month after portfolio formation and follow the methodology of Jegadeesh and Titman (1993) to report momentum profits in the next 6-month holding period. "Low EDF" refers to the bottom EDF quintile and "High EDF" to the top quintile. *AVL* is asset size, *R&D* is the ratio of R&D expenditures over total book assets, and *SalesHfdl* is the Herfindahl index for sales within an industry. Portfolio returns are expressed in percentage per month. $W - L$ are the raw momentum profits, FF-α and 4-Factor α refer to momentum profits after controlling for risk according to the Fama–French three-factor model and the Carhart four-factor model, respectively.

	Low EDF						High EDF					
	W−L	t-stat	FF-α	t-stat	4-Factor α	t-stat	W−L	t-stat	FF-α	t-stat	4-Factor α	t-stat
Panel A: Momentum profits across *AVL* groups												
Low	1.39	3.599	1.49	3.780	0.93	2.372	0.79	2.784	0.84	2.882	0.00	−0.001
Med	1.22	5.130	1.39	5.795	0.57	3.065	1.64	4.687	1.76	4.893	0.67	2.227
High	0.73	2.814	0.84	3.117	−0.25	−1.466	1.90	3.767	2.03	3.940	0.79	1.685
High−Low	−0.66	−1.605	−0.66	−1.605	−1.18	−2.799	1.10	2.330	1.20	2.446	0.79	1.597
Panel B: Momentum profits across *R&D* groups												
Low	0.81	2.304	0.92	2.560	−0.17	−0.557	2.00	4.570	2.19	4.877	1.05	2.608
Med	1.10	3.486	1.16	3.532	0.16	0.570	0.47	1.099	0.51	1.170	−0.55	−1.384
High	0.65	1.857	0.86	2.381	−0.05	−0.141	0.26	0.455	0.61	1.051	−0.55	−0.991
High−Low	−0.16	−0.369	−0.07	−0.150	0.12	0.267	−1.75	−2.951	−1.58	−2.601	−1.60	−2.553
Panel C: Momentum profits across *SalesHfdl* groups												
Low	0.85	2.801	0.87	2.772	−0.16	−0.630	1.20	3.089	1.23	3.082	0.15	0.434
Med	0.72	2.554	0.77	2.667	−0.15	−0.624	1.01	2.656	1.17	2.992	0.20	0.572
High	0.75	2.441	0.96	3.053	−0.14	−0.608	2.14	5.302	2.27	5.497	1.21	3.296
High−Low	−0.10	−0.308	0.10	0.299	0.01	0.035	0.94	2.234	1.04	2.393	1.06	2.379

Panel A of Table IV shows that for firms with high EDF scores (subpanel labeled “High EDF”) and large asset bases, winners outperform losers by 1.9% per month over the next 6-month period. Among those high EDF firms with small asset bases, past winners outperform past losers by 0.79% per month. This difference of 1.1% per month is significant at the 5% confidence level. This pattern persists when we examine the Fama–French three-factor-adjusted alpha for momentum profits, which yields a difference of 1.2% per month between large firms and small firms. When we add the momentum factor to the Fama–French adjustment, the alpha for firms with small asset values becomes zero, indicating that there is no enhanced momentum profit for these firms. For large firms, however, there is still a sizeable enhanced momentum profit of 0.79% per month. These results are consistent with our prediction regarding the role of shareholder recovery in producing enhanced momentum profits among stocks with high default probabilities. Notice that for firms with low EDF scores none of these patterns is present, as shown in the left-hand side of Panel A. In fact, asset size has an opposite effect on both raw and risk-adjusted momentum profits, that is, enhanced momentum profits are weaker for large firms. This finding is consistent with the argument of Sagi and Seasholes (2007), who attribute momentum to growth options in absence of financial leverage, usually associated with small firms.

Panel B of Table IV demonstrates that firms with high EDF scores and low R&D expenditures experience strong momentum in stock returns, but firms with high R&D expenditures and similar credit profiles do not. Again, this pattern is robust to the risk adjustment according to both the Fama–French three-factor model and the momentum-augmented four-factor model. Furthermore, none of these patterns is present among firms with low EDF scores, as the left-hand side of Panel B illustrates. In Panel C, we test whether the liquidation costs aspect of shareholder recovery has an impact on momentum profits. As the panel shows, high EDF firms in a more concentrated industry, that is, high *SalesHfdl* portfolios, have stronger momentum in stock returns than similar firms in a more competitive industry, and the statistical significance of this difference remains strong despite the coarse nature of the Herfindahl measure. This pattern is even stronger after risk adjustment, but it disappears when we restrict our analysis to low EDF firms. In summary, the evidence presented in Table IV strongly supports the prediction of our model regarding the importance of shareholder recovery for financially distressed firms in enhancing the profitability of momentum strategies.

Our large database of expected default frequencies also allows us to provide a comprehensive picture of the relationship between momentum profits and default probability. In their study, Avramov et al. (2007) rely on a sample of stocks for which a firm-level S&P credit rating is available. This sample represents a small subset of the entire cross-section of stocks. In contrast, our sample of stocks with an EDF measure covers virtually all publicly traded stocks in the CRSP database. To better understand the relationship between momentum profits and default probability, in Table V we report both the unconditional momentum profits (“Uncond.”) and the momentum profits conditional

Table V

Momentum Profits and Default Probability

The column labeled “Uncond.” reports momentum profits computed according to the “6-1-6” procedure in Jegadeesh and Titman (1993). The remaining columns report momentum profits similarly computed within EDF quintiles. To obtain these values, each month all stocks are sorted independently into quintiles of EDF scores and quintiles of winners/losers according to past 6-month returns. We skip a month after portfolio formation. The value-weighted returns of each portfolio for the subsequent 6-month period are recorded and averaged over time. Portfolio returns are expressed in percentage per month. Momentum alphas are obtained after controlling for risk according to the Carhart four-factor model.

		EDF					
	Uncond.	1	2	3	4	5	Diff
Raw profits	0.80	0.70	0.70	0.97	1.06	1.54	0.84
t-stat	3.145	2.772	2.830	4.301	4.309	4.780	2.715
4-Factors alphas	−0.23	−0.28	−0.32	0.09	0.17	0.52	0.80
t-stat	−2.022	−1.675	−2.109	0.588	0.956	2.042	2.463
			Factor loadings				
UMD	1.186	1.049	1.050	0.905	0.934	1.094	0.045
t-stat	45.313	27.466	29.422	25.314	22.670	18.378	0.596
MKT	−0.045	0.013	0.058	0.042	−0.021	−0.013	−0.027
t-stat	−1.664	0.337	1.578	1.128	−0.486	−0.218	−0.343
HML	0.185	0.206	0.223	0.209	0.241	0.295	0.089
t-stat	4.594	3.502	4.061	3.792	3.802	3.213	0.763
SMB	−0.092	0.102	0.227	0.145	0.146	−0.090	−0.192
t-stat	−2.654	2.021	4.785	3.060	2.675	−1.138	−1.924

on the level of EDF. The unconditional momentum profit is 0.8% per month, statistically significant at the 1% level and comparable in magnitude to the evidence documented in prior literature. Interestingly, adjusting for risk within a four-factor model—which includes the momentum factor (UMD)—produces a negative alpha of −0.23%, statistically significant at the 5% level. The factor loadings indicate that a large portion of the momentum profits is accounted for by the momentum factor.

The remaining columns of Table V show that the conditional momentum profits range from 0.7% a month in the lowest EDF quintile to 1.54% a month in the highest EDF quintile. The difference of 0.84% per month is both economically and statistically significant. When we adjust momentum profits with the four-factor model, the pattern persists. In fact, for low EDF quintiles the alpha turns negative, but it increases to 0.52% per month for the highest EDF quintile. This is consistent with our prediction that increased distress risk contributes to enhanced momentum profits that are not captured by the momentum factor. Moreover, the table also shows that although momentum profits load strongly on the momentum factor with a factor loading close to one, they are almost orthogonal to the market factor. Their loadings on the HML factor are largely uniform across EDF quintiles.

One striking observation, however, is that for low EDF quintiles, momentum profits load positively on the SMB factor, whereas for the highest EDF quintile, the loading becomes negative, albeit statistically insignificant.[23] Moreover, the difference between SMB loadings in the top and bottom quintiles is statistically significant at the 5% level. The implication of this finding is that for lightly levered firms, small stocks drive the enhanced momentum strategy, whereas for heavily levered firms, large stocks contribute more to the enhanced momentum profits. Therefore, for low EDF firms, growth options matter as in Sagi and Seasholes (2007), whereas for high EDF firms, potential shareholder recovery, which is more likely for larger firms, plays an indispensable role in producing enhanced momentum profits, as predicted by our model.

III. Conclusion

Recent empirical evidence strongly suggests that financial distress is instrumental in understanding cross-sectional properties of stock returns. Although this seems to confirm the conjecture of Fama and French (1992) that the book-to-market effect and other cross-sectional "anomalies" are related to the risk of financial distress, efforts toward finding a distress risk factor have unveiled puzzling empirical patterns.

In this paper, we propose a new perspective for understanding the empirical regularities in the cross-section of equity returns. We explicitly introduce financial leverage in a simple equity valuation model and investigate how the possibility of shareholder recovery upon financial distress affects the relationship between a firm's expected return and its likelihood of default. Within this simple framework, we derive three important insights.

First, the presence of potential shareholder recovery upon financial distress alters the risk structure of equity and causes the equity beta and expected returns to be *hump shaped* in default probability. Second, this nonmonotonic relationship between risk and default probability leads to hump-shaped value spreads with respect to default probability. Third, the same hump-shaped relationship between expected returns and default probability predicts that momentum profits should be enhanced among firms with both high default likelihood and strong prospects for shareholder recovery upon financial distress. These predictions are robust in a general model with endogenous investment and financing choices.

Using the EDF from MKMV as a market-based measure of default probability, we empirically confirm the hump-shaped relationship between equity beta and default probability and find support for the novel predictions of our theory in the data. Specifically, the value premium is hump shaped in default probability and momentum profits are stronger for stocks with a higher likelihood of default and larger prospects for shareholder recovery. These results complement and corroborate recent empirical evidence on patterns of conditional betas that are consistent with the predictions of our model.

[23] The results based on equal-weighted returns are reported in the Internet Appendix. Overall, the patterns are similar to the value-weighted results of Table V, but the SMB loading is negative and statistically significant for the highest EDF quintile, similar in magnitude to that of unconditional momentum profits in the table.

At a more general level, the perspective we offer underscores the importance of financial leverage and the resolution of financial distress in asset pricing models for levered equity. Our model highlights the role of shareholder recovery upon financial distress as a pervasive mechanism for understanding cross-sectional variation in both the value premium and momentum profits. Although in this paper we focus our attention on these two major cross-sectional regularities in equity returns, our framework promises to be a useful platform for understanding a broader set of cross-sectional properties of both stock and bond returns.

Appendix A. Proofs

Proof of Proposition 1: Consider first the value of an unlevered firm $V^U(p_t)$ that has no debt and operating costs and receives a continuous stream of cash flow, e^{p_t}. From (2), the value of this firm is

$$V^U(p_t) = \int_0^\infty E^{\boldsymbol{Q}}\left[e^{p_{t+\tau}}\right] d\tau = \int_0^\infty e^{-r\tau} \left[\exp\left(p_t e^{-\zeta\tau} + \left(1 - e^{-\zeta\tau}\right)\frac{\mu^{\boldsymbol{Q}}}{\zeta} + \sigma^2 \frac{1 - e^{-2\zeta\tau}}{4\zeta}\right)\right] d\tau.$$

Now consider the equity value of a firm that issues debt with perpetual coupon l and faces operating costs of c per unit of time. Ignoring the limited liability option, the equity value $V^L(p_t)$ of this firm is

$$V^L(p_t) = V^U(p_t) - \frac{l + c}{r}. \tag{A1}$$

The value $V(p_t)$ of equity with limited liability is given by $V(p_t) = V^L(p_t) + U(p_t)$, where $U(p_t)$ represents the value of the limited liability option. The value of $U(p_t)$ satisfies the following ordinary differential equation:

$$\frac{1}{2}\sigma^2 U''(p) + (\mu^{\boldsymbol{Q}} - \zeta p)U'(p) - rU(p) = 0, \tag{A2}$$

whose solution is given by

$$U(p_t) = A \cdot H\left(-\frac{r}{\zeta}, -\frac{\mu^{\boldsymbol{Q}} - \zeta p_t}{\sigma\sqrt{\zeta}}\right) + B \cdot H\left(-\frac{r}{\zeta}, \frac{\mu^{\boldsymbol{Q}} - \zeta p_t}{\sigma\sqrt{\zeta}}\right), \tag{A3}$$

where $H(v, z)$ is the generalized Hermite function of order v (see, e.g., Abramowitz and Stegun (1972)):

$$H(v, z) = \frac{2^{v+1}}{\sqrt{\pi}} e^{z^2} \int_0^\infty e^{-t^2} t^v \cos\left(2zt - \frac{\pi v}{2}\right) dt. \tag{A4}$$

Imposing the boundary condition $\lim_{p_t \to \infty} U(p_t) = 0$ allows us to exclude the second Hermite function in (A3). Upon default, $p_t \leq \underline{p}(\eta)$, the equity holder recovers a fraction η of the residual value $R(p_t)$. Therefore, the value of the firm's equity can be written as in (5), where the constants A and $\underline{p}(\eta)$ are obtained from the value-matching and smooth-pasting conditions (7) and (8). $\square$

LEMMA 1 (DEFAULT PROBABILITY UNDER THE PHYSICAL MEASURE): *Let p_0 be the current value of the product log price, evolving according to the process described in (1) with $\zeta \to 0$, and let $\underline{p}$ be the endogenously determined default trigger. The time 0 cumulative real default probability* $\Pr_{(0,T]}$ *over the time period* $(0, T]$ *is given by*

$$\Pr_{(0,T]}(p_0) = \mathcal{N}\left(h(T)\right) + e^{-\frac{2\omega}{\sigma^2}(p_0 - \underline{p})}\mathcal{N}\left(h(T) + \frac{2\omega T}{\sigma\sqrt{T}}\right), \tag{A5}$$

with $\omega = \mu - \frac{1}{2}\sigma^2 > 0$, $h(T) = \frac{\underline{p} - p_0 - \omega T}{\sigma\sqrt{T}}$, *and* $\mathcal{N}(\cdot)$ *the cumulative standard normal function.*

Proof of Lemma 1: The lemma is proved via direct application of the hitting time distribution of a Brownian motion; see, for example, Harrison (1985), equation (11), p. 14. $\square$

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